

# Contract cost in central banking: treating the symptom vs. treating the disease?

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## Abstract

Chortareas and Miller (2003) extended the basic principal-agent framework for central banking to consider that contracts generate costs for the government. In such setting, they claim that an output contract yields higher social welfare than an inflation contract, since only the former eliminates the inflation bias. We challenge these conclusions and prove that both incentive schemes (and any other generic linear output or inflation contract) eliminate this bias, achieving the same social welfare, provided the government behaves in accordance with optimizing behavior. Finally we discuss how the assumptions in C-M (2003) may be changed so as to salvage their results.

Keywords: central bank, inflation bias, contract cost

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# 1 Introduction

Central bank independence is a well-known remedy to the inflation bias arising to the time-inconsistency problem of discretionary monetary policy. Walsh (1995) modelled this process of delegation as a contract within a principal-agent framework. He showed that the inflation bias can be eliminated if the government (principal) offers the central bank (agent) an inflation contract which penalizes the latter for creating inflation. He established this result in a setting where such incentive transfer does not generate a cost for the government. In other words, this conclusion was obtained in a setting where the government's objective is solely to minimize the expected loss, measured by the deviations of output and inflation from socially desirable levels.

However, Walsh (1995, p. 156, ft. 10) also stated that “An alternative approach would assume that the government's objective is to minimize the expected loss plus the transfer to the central banker. However, in the present context, the government will always be able to offer a minimum-cost contract. Therefore, assuming that the government minimized  $E(V)$  (*expected loss*) instead of  $E(V+t)$  (*expected loss plus the transfer*) involves no loss of generality” (*words in italics added*).

Chortareas and Miller (2003) in what follows C-M (2003), extended the model in this direction, namely, considering that contracts generate costs for the government. They claimed the following two main conclusions which we challenge in our paper:

(C1) A linear inflation contract cannot completely eliminate the inflation bias (Proposition 1, p. 107).

(C2) An output contract achieves higher welfare, relative to an inflation contract (This conclusion is obtained upon comparing their Proposition 1 with their Proposition 2, p. 110).

Notice that (C1) amounts to stating that the Walsh's claim (referred to in our second paragraph) is inaccurate. In fact, referring to this claim C-M (2003) state: “As we demonstrate, this conjecture is inaccurate” (p. 102, ft. 3). On the other hand, they justify (C2) by making use of the following “seemingly intuitive” but flawed metaphor: “the output contract treats the disease; the inflation contract treats the symptom”.

In a subsequent paper Chortareas and Miller (2007), in what follows C-M (2007), have contradicted

(C1)<sup>1</sup>. In this respect, upon reading C-M (2007) one may be misled into believing that the conclusions in both papers are different because they do not share the same model. However, we show that the true reason why results in both articles do not coincide lies in that C-M (2003) solve the model in an inappropriate way.

On the other hand, we also show that the conclusion (C2) cannot be derived from the model shared in those two papers. Moreover, C-M (2007) maintain as valid this conclusion (C2) and keep on making use of the same flawed metaphor which initially appeared in C-M (2003): “an output contract can completely eliminate the inflation bias by addressing the ‘cause of the disease’, the output bias, rather than the ‘symptom’ of the disease, ‘the excess inflation outcome’” (p. 244, ft. 1).

It should be emphasized at the very outset that our paper shares with C-M (2003) and (2007) the following assumption (also appearing in Walsh, 1995, p. 156, first paragraph) which is crucial for the results that we discuss:

(A1) the participation constraint of the central banker (agent) is satisfied.<sup>2</sup>

In this sense, upon reading C-M (2007) one may have the wrong impression that they make an additional requirement or assumption, namely, that they impose that the participation constraint holds as an equality. For instance, they claim that their paper addresses the following question: “what happens if we require that the participation constraint holds with an equality?” (p.245, last paragraph). However, as we show below, this is not an additional “requirement” but an implication of the optimizing behavior of the principal. That is, if the participation constraint is assumed to be satisfied, it must hold as an equality in this setup (i.e., it cannot hold as an inequality). The reason is that, if the principal acts in accordance with optimization behavior, the so-called minimum-cost contract (in Walsh’s words) prevents the agent from obtaining any surplus in excess of the reservation utility level

To sum up, in this comment, we show that, under the assumptions in C-M (2003), the way by which C-M (2003) solve the model is inappropriate and, as a result, their conclusions (C1) and (C2) must be explicitly modified. On the other hand, we prove that, apart from the output contract proposed by

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<sup>1</sup>Candel-Sánchez and Campoy-Miñarro (2004, p. 35 and ft.4) also allude to this outcome.

<sup>2</sup>It is important that the reader bears in mind that this assumption is also made in C-M (2003). For instance, in C-M (2007) they state “Chortareas and Miller (2003) implicitly assume that the participation constraint holds ” (p. 244, ft. 1) or “Chortareas and Miller (2003) implicitly impose the participation constraint ” (p. 247).

C-M (2003) and the Walsh inflation contract, any generic contract which linearly links, explicitly or implicitly, the central bank's incentives to inflation eliminates the inflation bias and achieves the same level of social welfare, provided that it is designed in accordance with optimizing behavior. In this sense, this wide range of contracts are the ones that do "treat the disease" in an appropriate manner. Finally, we briefly discuss how the assumptions in C-M (2003) may be changed so as to salvage their results.

## 2 The model

As in C-M (2003), and following their notation for ease of comparison, the working of the economy is summarized by the following equations<sup>3</sup>:

$$y = y^n + \alpha(\pi - \pi^e) + \varepsilon, \quad (1)$$

$$\pi = m + \nu - \gamma\varepsilon, \quad (2)$$

$$U^G = - \left[ (y - y^*)^2 + \beta\pi^2 \right] - \phi [tr(.)], \quad (3)$$

$$U^{CB} = - \left[ (y - y^*)^2 + \beta\pi^2 \right] + \xi [tr(.)], \quad (4)$$

where  $y^n$ ,  $\alpha$ ,  $\beta$ ,  $\phi$ ,  $\xi > 0$  and  $k \equiv y^* - y^n > 0$ ; and superscripts, "G" and "CB", respectively, stand for "Government" and "Central Bank". Equation (1) shows that the economy possesses a Lucas supply function, so that the difference between output ( $y$ ) and the natural level ( $y^n$ ) depends on the deviations of inflation ( $\pi$ ) from its rationally expected value ( $\pi^e$ ) and on a supply shock ( $\varepsilon$ ) with zero mean and finite variance ( $\sigma_\varepsilon^2$ ). Expression (2) states that inflation is a function of: a) the growth of a monetary aggregate determined by the central bank ( $m$ ); b) a velocity shock or a control error ( $\nu$ ), with  $E\{\nu\} = 0$  and  $E\{\nu^2\} = \sigma_\nu^2$ , which is uncorrelated with  $\varepsilon$ ; and c) the supply shock (which also appears in (1)), where  $\gamma$  picks up the effect of this shock on inflation.

Equations (3) and (4) represent the utility functions of, respectively, the government (principal) and central bank (agent). Each of these two expressions consists of two terms. The corresponding first terms mean that the government and the central bank care about deviations of inflation and output

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<sup>3</sup>The framework is the one considered by Walsh (1995), except for that C-M (2003) extend it to consider that incentive schemes represents a cost for the government. Notice that the same model is used in C-M (2007) with the only exception that this paper does not consider a control error. This minor difference does not affect the results.

from some desired levels. This term is identical for both of these players and their common target value for inflation is normalized to zero. The second terms in (3) and (4) stand for the valuation that the principal and the agent have, respectively, of the incentive scheme ( $tr(.)$ ) designed by the former, which may take the form of an inflation or an output contract. Besides, the government is assumed to have the social preferences. Therefore, social welfare will be evaluated in terms of the government's expected utility function.

The interactions between the government, the central bank and the private sector are modelled through a multi-stage game. The sequence of events is as follows: a) The government offers the central bank a contract ( $tr(.)$ ); b) the private sector observes the incentive scheme and then forms its expectations on inflation ( $\pi^e$ ); c) The realization of the output shock ( $\varepsilon$ ) becomes common knowledge; d) The central bank selects the level of the policy instrument ( $m$ ); e) The stochastic control error or velocity shock takes place ( $\nu$ ).

### 3 The results

#### 3.1 Inflation contract

We begin by analyzing the case where the government offers the central bank a linear inflation contract. Formally, the incentive scheme designed by the principal is:

$$tr(.) = t_0 - t\pi. \quad (5)$$

We look for a subgame perfect equilibrium. Therefore, we apply backward induction to the game outlined in Section 2 (for the case of an inflation contract). In the last stage of the game, the central banker selects the value for  $m$  that solves the following program (bearing in mind (1), (2), (3) and (5)):

$$\begin{aligned} \underset{\{m\}}{Max} & - \left[ (y - y^*)^2 + \beta\pi^2 \right] + \xi [t_0 - t\pi] \\ s.t. & \begin{cases} y = y^n + \alpha(\pi - \pi^e) + \varepsilon, \\ \pi = m + \nu - \gamma\varepsilon. \end{cases} \end{aligned}$$

Solving and taking expectations yields the expressions for expected and actual inflation:

$$\pi^e = \frac{\alpha}{\beta}k - \frac{\xi}{2\beta}t. \quad (6)$$

$$\pi = \frac{\alpha}{\beta}k - \frac{\xi}{2\beta}t + \nu - \left(\frac{\alpha}{\alpha^2 + \beta}\right)\varepsilon \quad (7)$$

Our analysis so far has been equivalent to the one by C-M (2003). However, the way in which we solve the first stage of the game is completely different from theirs. In order to find the solution of the first stage, we need to express the expected utility functions of the government and the central bank in terms of the variables which define the contract, namely,  $t_0$  and  $t$ . Therefore we modify equations (3) and (4) following this sequence of computations: i) substituting  $tr(\cdot) = t_0 - t\pi$  and setting and  $k \equiv y^* - y^n > 0$ ; ii) plugging (1); iii) substituting the values for  $\pi^e$  and  $\pi$  (appearing in equations (6) and (7)); and, finally, iv) taking expectations. This yields:

$$E(U^G) = -\phi t_0 - \frac{(2\phi + \xi)\xi}{4\beta}t^2 + \frac{\alpha k(\xi + \phi)}{\beta}t - (\alpha^2 + \beta)\left(\frac{k^2}{\beta} + \sigma_\nu^2\right) - \frac{\beta}{\alpha^2 + \beta}\sigma_\varepsilon^2, \quad (8)$$

$$E(U^{CB}) = \xi t_0 + \frac{\xi^2}{4\beta}t^2 - (\alpha^2 + \beta)\left(\frac{k^2}{\beta} + \sigma_\nu^2\right) - \frac{\beta}{\alpha^2 + \beta}\sigma_\varepsilon^2. \quad (9)$$

In the first stage, the principal chooses the value of its strategic variables, namely, the ones that shape the contract. It does so bearing in mind that the monetary authorities must accept the incentive scheme being offered. This ‘‘participation constraint’’ states that the expected utility obtained by the central bank when signing the contract must be higher or equal to a given reservation level, normalized to zero. Therefore, the government solves:

$$\begin{aligned} \underset{\{t_0, t\}}{Max} \quad & E(U^G) \\ \text{s.t.} \quad & E(U^{CB}) \geq 0, \end{aligned}$$

Solving the initial two Kuhn-Tucker first order conditions ( $\frac{\partial \mathcal{L}}{\partial t_0} = \frac{\partial \mathcal{L}}{\partial t} = 0$ ) for the Lagrangian multiplier,  $\mu$ , and equating yields:

$$\mu = -\frac{\frac{\partial E(U^G)}{\partial t_0}}{\frac{\partial E(U^{CB})}{\partial t_0}} = -\frac{\frac{\partial E(U^G)}{\partial t}}{\frac{\partial E(U^{CB})}{\partial t}}. \quad (10)$$

Now, rearranging we obtain the equality of the marginal rates of substitution (between  $t_0$  and  $t$ ) of the government and the central bank:

$$\left.\frac{\partial t_0}{\partial t}\right|_{E(U^G)=\overline{E(U^G)}} = \frac{\frac{\partial E(U^G)}{\partial t}}{\frac{\partial E(U^G)}{\partial t_0}} = \frac{\frac{\partial E(U^{CB})}{\partial t}}{\frac{\partial E(U^{CB})}{\partial t_0}} = \left.\frac{\partial t_0}{\partial t}\right|_{E(U^{CB})=\overline{E(U^{CB})}}. \quad (11)$$

Calculating both marginal rates of substitution (from (8), (9) and (11)) we have:

$$\left. \frac{\partial t_0}{\partial t} \right|_{E(U^G)=\overline{E(U^G)}} = \frac{\alpha k (\xi + \phi)}{\phi \beta} - \frac{\xi (2\phi + \xi)}{2\phi \beta} t, \quad (12)$$

$$\left. \frac{\partial t_0}{\partial t} \right|_{E(U^{CB})=\overline{E(U^{CB})}} = -\frac{\xi}{2\beta} t. \quad (13)$$

Equating (12) and (13) and rearranging yields the optimal penalization on inflation:

$$t^\pi = \frac{2\alpha k}{\xi} \quad (14)$$

Finally, the resulting inflation bias is determined by substituting (14) into (6), which yields an expected inflation equal to zero. In other words, the inflation contract completely eliminates this bias. This result is in sharp contrast with the one by C-M (2003) who, in their Proposition 1, claimed quite the opposite, i.e.: “Unless the government places a zero weight on the cost associated with the central banker’s incentive scheme, a linear inflation contract cannot completely eliminate the inflation bias”

*Remark 1: Given the assumptions in C-M (2003), their way of obtaining the penalization rate is inappropriate.*

This is the case since C-M (2003) failed to take account of the fact that the principal faces a constraint optimization problem when solving for the penalization rate ( $t$ ).<sup>4</sup> Or, which is equivalent in terms of calculus, C-M (2003) worked out this penalization rate as if the participation constraint of the agent held as an inequality, i.e., as if the Lagrangian multiplier were zero. However, this way of solving the model is inappropriate since this multiplier does take the following strictly positive value

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<sup>4</sup>That is, they solve for the value of the penalization rate as if the government were facing a free optimization problem, i.e., overlooking the participation constraint of the central bank. Formally, C-M (2003, p. 106) solve the following equation (first-order condition):

$$\frac{\partial E(L^G)}{\partial t} = 0,$$

which yields their claimed equilibrium value of  $t$  (appearing in their expression (15)). Notice also that C-M (2003) implicitly assume that the fixed term ( $t_0$ ) is a choice variable whose value is adjusted so that, taking into account the equilibrium value of  $t$ , the participation constraint holds as an equality.

Notice that if  $t_0$  were not a choice variable, then, taking as given this “exogenous” value, it would now be the value of the penalization rate the one that would be adjusted so that the participation constraint held as an equality. However, in such a case the way C-M (2003) solve for  $t$  (assuming a free optimization problem) would also be inappropriate.

(from (8)-(10))<sup>5</sup>:

$$\mu = \frac{\phi}{\xi} > 0 \quad (15)$$

The reason why the participation constraint is binding is that, otherwise, the principal would not be behaving optimally since it would be better-off by lowering the fixed part of its contract ( $t_0$ ) so that the central bank still accepted it.<sup>6</sup>

To sum up, when the incentive scheme involves a cost to the government, as assumed by C-M (2003) (i.e.,  $\phi > 0$  in (3)), the agent's participation constraint has to be taken into account (and treated as an equality) to solve for the penalization rate of the inflation contract. In other words, in this setting, the principal's program cannot be solved as a free optimization problem. As a result, the contract obtained by C-M (2003) is not derived from an optimizing behavior from the part of the principal and cannot eliminate the inflation bias.<sup>7</sup>

Notice that, upon reading C-M (2007) one may have the wrong impression that they make an additional requirement or assumption, namely, that they impose that the participation constraint holds with equality. But, as we have shown, this is not an additional "requirement". The reason is that the assumption that the participation is satisfied coupled with the requirement that the principal acts optimally implies that the participation constraint must hold as an equality. Otherwise, the principal would pay the agent an "excessive" incentive transfer (in excess of the reservation utility level), i.e., the incentive scheme offered would not a minimum-cost contract.

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<sup>5</sup>Recall that the Kuhn-Tucker conditions of a maximization problem subject to a constraint imply that: (a) when the Lagrangean multiplier is positive, then the constraint holds as an equality (i.e., it is binding); (b) and when this multiplier is zero the constraint holds as an inequality (i.e. it is not binding) and can be ignored.

<sup>6</sup>Notice that in the Walsh's (1995) setting, i.e., when the government does not care about the cost of the contract ( $\phi = 0$ ) the Lagrangian multiplier is zero (i.e., setting  $\phi = 0$  in (15) yields  $\mu = 0$ ). In this specific case, the participation constraint is not binding (i.e., the government does not care if the fixed part of the contract is so high that the central banker gets a surplus in excess of the reservation utility level) and, therefore, can be ignored. For this reason, Walsh (1995), solves the penalization rate in a non-constraint setting and sets  $t_0$  at a sufficiently high level that insures that the participation constraint is satisfied.

<sup>7</sup>For the same reason, Corollary 2 in C-M (2003) does not hold either. That is, their claim that "the more 'cost-conscious' the government is, the lower the required marginal penalization rate is in the government's optimal contract" is not in accordance with our expression (14), i.e., the marginal penalization rate ( $t^\pi$ ) does not depend on the government's degree of cost-consciousness ( $\phi$ )

### 3.2 Output contract

In this subsection we take up the scenario where the incentive scheme is:

$$tr = t_0 - t(y - y^n). \quad (16)$$

The game is solved in an analogous way as the one described in the previous subsection. Therefore, in the last stage, central bank solves (from (1), (2), (4) and (16)):

$$\begin{array}{l} \underset{\{m\}}{Max} \\ s.t. \end{array} \quad \begin{cases} - \left[ (y - y^*)^2 + \beta\pi^2 \right] + \xi [t_0 - t(y - y^n)] \\ y = y^n + \alpha(\pi - \pi^e) + \varepsilon, \\ \pi = m + \nu - \gamma\varepsilon, \end{cases}$$

Expected and actual inflation are obtained by solving this problem and making use of the expectation operator:

$$\pi^e = m^e = \frac{\alpha k}{\beta} - \left( \frac{\xi \alpha}{2\beta} \right) t, \quad (17)$$

$$\pi = \frac{\alpha k}{\beta} - \frac{\xi \alpha}{2\beta} t - \left( \frac{\alpha}{\alpha^2 + \beta} \right) \varepsilon + \nu. \quad (18)$$

*Proposition 1: Social welfare is the same irrespective of whether the central bank is offered an inflation contract or an incentive scheme which penalizes it when output exceeds its natural level.*

*Proof:*

1) We begin by considering the case of an output contract. In this context, we move up to the first stage. Therefore, we need to express the expected utility functions of the government and the central bank in terms of the choice variables,  $t_0$  and  $t$  that determined the output contract. Therefore, making use of the expectation operator together with (1), (3), (4), (17) and (18) yields:

$$E(U^G) = -\phi t_0 - \frac{\alpha^2 \xi^2}{4\beta} t^2 + \frac{\alpha^2 k \xi}{\beta} t - (\alpha^2 + \beta) \left( \frac{k^2}{\beta} + \sigma_\nu^2 \right) - \frac{\beta}{\alpha^2 + \beta} \sigma_\varepsilon^2, \quad (19)$$

$$E(U^{CB}) = \xi t_0 - \frac{\alpha^2 \xi^2}{4\beta} t^2 + \frac{\alpha^2 k \xi}{\beta} t - (\alpha^2 + \beta) \left( \frac{k^2}{\beta} + \sigma_\nu^2 \right) - \frac{\beta}{\alpha^2 + \beta} \sigma_\varepsilon^2. \quad (20)$$

Again, making use of the Kuhn-Tucker conditions in the same way as in the previous subsection, one obtains the marginal rates of substitution between  $t_0$  and  $t$  for, respectively, the government and the

central bank:

$$\left. \frac{\partial t_0}{\partial t} \right|_{E(U^G)=\overline{E(U^G)}} = \frac{\xi (2k - \xi t) \alpha^2}{2\phi\beta}, \quad (21)$$

$$\left. \frac{\partial t_0}{\partial t} \right|_{E(U^{CB})=\overline{E(U^{CB})}} = -\frac{(2k - \xi t) \alpha^2}{2\beta}. \quad (22)$$

Equating both marginal rate of substitution one obtains the optimal penalization on inflation:

$$t^y = \frac{2k}{\xi}. \quad (23)$$

The inflation bias is also eliminated with this output contract since substituting (23) into (17) yields that expected inflation is equal to zero as well.

Upon substituting (23) into (20) and equating to zero (since, again, the participation constraint must hold with equality) the equilibrium value of the fixed part of the contract is found to be:

$$t_0^y = \frac{1}{\xi} \left( k^2 + (\alpha^2 + \beta) \sigma_\nu^2 + \frac{\beta \sigma_\varepsilon^2}{(\alpha^2 + \beta)} \right). \quad (24)$$

The expected utility for society with this optimum contract is (plugging (23) and (24) into (19)):

$$E(U_y^G) = -\frac{(\xi + \phi) k^2}{\xi} - \frac{(\alpha^2 + \beta) (\xi + \phi) \sigma_\nu^2}{\xi} - \frac{\beta (\xi + \phi) \sigma_\varepsilon^2}{(\alpha^2 + \beta) \xi} \quad (25)$$

2) Now, we calculate the expected utility of society under an inflation contract. Since the associated Lagrangian multiplier appearing in (15) is strictly positive, in this case the participation constraint also holds with equality. Therefore, equating the expected utility of the central banker (given in (9)) to its reservation level, plugging the value for the penalization rate (appearing in (14)) into the resulting equation and solving for  $t_0$ , one obtains the expression for the equilibrium value of the fixed part of the inflation contract:

$$t_0^\pi = \frac{1}{\xi} \left( k^2 + (\alpha^2 + \beta) \sigma_\nu^2 + \frac{\beta}{(\alpha^2 + \beta)} \sigma_\varepsilon^2 \right). \quad (26)$$

Plugging (14) and (26) into (9) one finds the principal's expected utility under an optimum inflation contract:

$$E(U_\pi^G) = -\frac{(\xi + \phi) k^2}{\xi} - \frac{(\alpha^2 + \beta) (\xi + \phi) \sigma_\nu^2}{\xi} - \frac{\beta (\xi + \phi) \sigma_\varepsilon^2}{(\alpha^2 + \beta) \xi} \quad (27)$$

3) Finally, notice that this value of expected social utility under a inflation contract coincides with the one in the case where the government designs an optimal output contract (expression (25)).

Therefore, social welfare is the same irrespective of whether the central bank is offered an inflation contract or an output contract. ■

*Remark 2: This result is in sharp contrast with a main conclusion by C-M, namely, that when incentive schemes generate a cost for the government, it (strictly) prefers to offer the central bank an output contract, instead of an inflation contract.*

In this respect it is worth noting that the intuition provided by C-M to support their conclusion is thoroughly flawed. To wit, these authors claim that the reason why only their proposed output contract can remove the inflation bias is because it attacks directly the root of the expansionary bias for output while the inflation contract only attacks the consequence of this expansionary bias, i.e., the inflation. That is C-M (2003) make use of the following “seemingly intuitive” metaphor: “the output contract treats the disease; the inflation contract treats the symptom”.<sup>8</sup> We have shown that this explanation is not appropriate. Moreover, the following proposition generalizes the set of contracts

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<sup>8</sup>Note that, (only) in the case of an output contract, C-M (2003) are fortunate enough to end up proposing an incentive scheme which happen to coincide with the optimal one, even though their way of proceeding is, again, inappropriate. More precisely, C-M (2003) obtain the penalization rate by solving for  $t$  the following first order condition of a sort of “unconstrained” optimization problem from the part of the principal (i.e., overlooking the participation constraint of the agent):

$$\frac{\partial E(U^G)}{\partial t} = 0.$$

They do so instead of purposely taking into account the tangency condition between the isoexpected utilities curves of the government and the central bank in the  $(t_0, t)$  space, i.e. (see equation (11)):

$$\left. \frac{\partial t_0}{\partial t} \right|_{E(U^G)=\overline{E(U^G)}} = \left. \frac{\partial t_0}{\partial t} \right|_{E(U^{CB})=\overline{E(U^{CB})}}.$$

However, the value of  $t$  that fulfills this condition turns out to be, by coincidence, the same as the one obtained by C-M (2003) (i.e.  $t^y = 2\frac{k}{\xi}$ ). The reason is that, for this value of  $t$ , we have that:

$$\left. \frac{\partial E(U^G)}{\partial t} \right|_{t=t^y} = \left. \frac{\partial E(U^{BC})}{\partial t} \right|_{t=t^y} = 0.$$

To sum up, when trying to obtain the optimal penalization rate, C-M (2003) do make the very same analytical mistake incurred by them when they look for the optimal penalization on inflation (shown in the previous subsection). Namely, they overlook the participation constraint of the agent. In this respect it is well-known that solving a constraint optimization problem ignoring any possible constraint for all the choice variables involved implies that obtaining the correct solution of the problem is not warranted, namely, it can only happen by chance.

that can achieve the social optimum. In this sense, this type of incentive schemes are the ones that can be claimed to really “treat the disease” in an appropriate way:

*Proposition 2: Consider a “generic” linear contract of the form  $tr = t_0 + t(x + r)$ , where  $x$  is either output or inflation and  $r$  take any real number (positive, negative or zero). This generic incentive scheme yields the same level of social welfare provided the values of  $t_0$ , and  $t$  are chosen by the principal in accordance with optimizing behavior.*

*Proof:*

Denoting by  $t_0^g$  and  $t^g$ , respectively, the equilibrium values of  $t_0$  and  $t$  for this generic contract, it is straightforward to check that:

a) For the case of an output contract (i.e.,  $x = y$ ) we have that:

$t^g = -t^y$  and  $t_0^g = t_0^y + t^y(y^n + r)$ , where  $t^y$  and  $t_0^y$  appear, respectively, in (23) and (24).

b) When there is an inflation contract in place (i.e.,  $x = \pi$ ) the following applies:

$t^g = -t^\pi$  and  $t_0^g = t_0^\pi + t^\pi r$ , where  $t^\pi$  appears in (14) and  $t_0^\pi$  in (26). ■

Notice that when this generic incentive scheme is an output contract (i.e.,  $x = y$ ), the constant  $r$  need not be equal to  $-y^n$ , as in C-M (2003), namely, it may take any arbitrarily chosen real number (positive, negative or zero). In other words, penalizations need not be linked, to deviations of output from its natural level. As a consequence, the conclusion by C-M (2003) that their proposed contract is the solution to the inflation bias since it “treats the disease” (i.e., it penalizes increases of output beyond the natural level) is misleading.

The reason why, carrying on with the same metaphoric medical jargon, the “generic pharmaceutical” described in Proposition 2 maximizes social welfare is as follows. Given the quadratic form of the first term of the social utility function (first term appearing in brackets in (3)), any incentive scheme which linearly links (explicitly or implicitly) incentives to inflation does succeed in eliminating the inflation bias. In the case of an inflation contract incentives are explicitly linear in inflation. However, when the contract is linear in output incentives are also (but implicitly) linearly linked to inflation. Why? because output, in turn, is linear in inflation (from (1)).

## 4 Conclusions

The literature on monetary policy has stressed the importance of institutional arrangements as a way out of the inflation bias generated by the classic time-inconsistency problem to discretionary monetary policy. Walsh (1995) showed that the government (principal) can offer the central bank an incentive scheme contingent upon realized inflation -a linear inflation contract- in such a way that the inflation bias is completely eliminated. He proved this result in a setting where such incentives schemes generate no cost for the government. Chortareas and Miller (2003), C-M (2003), have extended the principal-agent setting to consider the possibility that incentive schemes represent a cost for the principal. In such scenario, C-M (2003) have concluded that an output contract achieves higher welfare than an inflation contract. They claim that the superiority of such an output contract lies in that it can eliminate the inflation bias, as opposed to an inflation contract.

We have shown that this conclusion cannot be derived from the assumptions of their model. The reason is that their analysis fails to take into account the participation constraint of the agent when solving for the penalization rate of the contract. This is the case even though C-M (2003) “unambiguously” claim that this constraint is assumed to hold in their paper. In other words, the paper by C-M (2003) shares with ours the very same model and the very same assumptions but not the results.

C-M (2003) justify their claimed superiority of the output contract by providing a flawed intuition (reiterated in C-M [2007]) that makes use of the following metaphor: an output contract treats the disease (the expansionary bias for output), as opposed to the inflation contract which treats the symptom (inflation). We have proved that this explanation is inappropriate since both types of contracts are equivalent. Moreover, carrying on with the C-M (2003) medical metaphor, we have shown that any “generic pharmaceutical” (i.e., any generic contract) which linearly links, explicitly or implicitly, the central bank’s incentives to inflation can succeed in eliminating the inflation bias, therefore, achieving the same level of social welfare.

Finally, we briefly discuss how the original assumptions of the model could be changed so as to salvage the results obtained by C-M (2003). At first sight, one could be tempted to state that just by removing the assumption that the participation constraint is satisfied (and keeping the remaining

assumptions of the model), the results claimed by C-M (2003) hold. However, this would imply that the fixed part of the contract ( $t_0$ ) would be chosen to be minus infinite. The reason is that in the paper by C-M (2003) contracts are assumed to represent a cost to the principal. A way out of this unappealing result requires changing more assumptions. For instance, that the fixed part of the contract offered by the government ( $t_0$ ) is exogenous in the model.<sup>9</sup> However, a much deeper discussion about this (or an alternative) change of assumptions and its justification goes beyond the scope of this comment.

## 5 References

Candel-Sanchez, F and Campoy-Miñarro, J.C. (2004). Is the Walsh contract really optimal? *Public Choice*: 120: 29-39.

Chortareas, G. and S. Miller (2003): “Monetary policy delegation, contract costs and contract targets”, *Bulletin of Economic Research*, 55:1, 101-112.

Chortareas, G. and S. Miller (2007): “The Walsh contract for the central bankers proves optimal after all!”, *Public Choice*, 131: 243-247.

Walsh, C. (1995): “Optimal contracts for central bankers”, *American Economic Review*, 85, 150-167.

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<sup>9</sup>Notice that C-M (2003) assume that the fixed term ( $t_0$ ) is not exogenous but a choice variable. Otherwise, taking as given this “exogenous” value, it would now be the value of the penalization rate the one that would be adjusted so that the participation constraint held with equality (which is not done in C-M [2003]).